SOLVED ASSIGNMENT ANSWERS
(2023-2024)

MATHEMATICS
(311)

Tutor Marked Assignment

1. Answer any one out of the following questions.
Write the following sets in roster form

a) $A = \{x : x \text{ is an integer and } -3 \leq x < 7\}$

Sol- The elements of this set are $-3, -2, -1, 0, 1, 2, 3, 4, 5, 6$ only.

Therefore, it can be written in roster form as:

$A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

b) $B = \{x : x \text{ is a natural number less than } 6\}$

Sol- The elements of this set are $1, 2, 3, 4$ and 5 only.

Therefore, it can be written in roster form as:

$B = \{1, 2, 3, 4, 5\}$

OR

a) From the given figure, find $\tan P - \cot R$.

Sol- In ΔPQR

$PR^2 = PQ^2 + QR^2$ .... (Pythagoras Theorem)

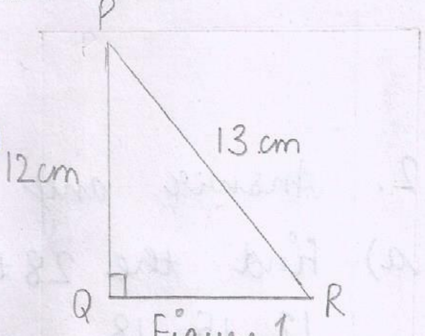

Figure 1

$13^2 = 12^2 + QR^2$

$QR^2 = \sqrt{169 - 144}$

$\sqrt{25} = 5$ cm

$\tan P = \frac{QR}{PQ} = \frac{5}{12}$ $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

$\tan P - \cot R = 0$

b) Prove that $(\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$.

Sol- L.H.S. -

$= (\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta$ ...... $[a^2 - b^2 = (a+b)(a-b)]$

$= [(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) + 1]\operatorname{cosec}^2\theta$

$= (\sin^2\theta - \cos^2\theta + 1)\operatorname{cosec}^2\theta$ ...... $[\sin^2\theta + \cos^2\theta = 1]$

$= 2\sin^2\theta \operatorname{cosec}^2\theta$ ...... $[1 - \cos^2\theta = \sin^2\theta]$

$= 2\cancel{\sin^2\theta} \cdot \frac{1}{\cancel{\sin^2\theta}}$

$= 2$

L.H.S = R.H.S

Hence, proved.

2. Answer any one out of the following questions.

a) Find the 28th term from the end of the A.P. 6, 9, 12, 15, 18, ..., 102.

Sol- The AP is 6, 9, 12, 15, 18, ..., 102

$a_1 = 6 \qquad a_2 = 9 \qquad d = a_2 - a_1 = 9 - 6 = 3$

$a_n = a + (n-1)d$

$a_{28} = 6 + (28-1)3$

$a_{28} = 6 + 27 \times 3$

$a_{28} = 6 + 81$

$a_{28} = 87$

So, 28^{th} term from the end is equal to 87.

b) How many 3-digit numbers are divisible by 7?

Sol- 3-digit numbers which are divisible by 7

105, 112, 119 999

$$\text{Number of terms} = \frac{\text{Last term} - \text{First term}}{\text{common difference}} + 1$$

$$= \frac{999 - 105}{7} + 1$$

$$= \frac{894}{7} + 1$$

$$= 127 + 1 = 128$$

There are 128 3-digit numbers divisible by 7

OR

a) The sum of some terms of a GP is 315. It's first term is 5 and the common ratio is 2. Find the number of it's term and the last term.

Sol- Let there be n terms of GP.

$a = 5 \quad r = 2$

Sum of n terms = 315

$$S_n = \frac{a.(r^n - 1)}{r - 1}$$

$$315 = \frac{5(2^n - 1)}{2 - 1}$$

$$315 = \frac{5(2^n - 1)}{1}$$

$$\frac{315}{5} = 2^n - 1$$

$$63 = 2^n - 1$$

$$2^n = 63 + 1$$

$$2^n = 64$$

$$2^n = 2^6$$

$$n = 6$$

Therefore, last term $= a.\, r^{(n-1)}$

$$= 5.2^{(6-1)} = 5.2^5 = 5.32 = 160$$

Therefore, the number of terms is 6 and last term is 160.

b) Find the common ratio of a GP whose sum of infinite terms is 8 and its second term is 2.

Sol- $S_\infty = \frac{a}{1-r}$

Let a be first term and r be common ratio.

$S_\infty = 8$

SOLVED ASSIGNMENT ANSWERS (2023-2024)

$8 = \dfrac{a}{1-r}$ — (1)

second term $a_2 = 2$

$a_2 = a.r$

$2 = a.r$

$a = \dfrac{2}{r}$ — (2)

Substitute a into in eq (1).

$8 = \dfrac{\frac{2}{r}}{1-r}$

$8(1-r) = 2r$

$8 - 8r = 2r$

$10r = 8$

$r = \dfrac{8}{10} = \dfrac{4}{5}$

3. Answer any one out of the following questions :

a) If $z_1 = 2+8i$ and $z_2 = 1-i$, then find $\left|\dfrac{z_1}{z_2}\right|$.

Sol - $\dfrac{z_1}{z_2} = \dfrac{2+8i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{2+2i+8i+8i^2}{1-i^2}$

$= \dfrac{2+10i-8}{1+1}$

$= \dfrac{-6+10i}{2}$

$= -3 + 5i$

$\left|\frac{z_1}{z_2}\right| = \sqrt{(-3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34}$

b) Suppose $z = (2-i)^2 + \left[\frac{(7-4i)}{(2+i)}\right] - 8$, express z in the form of $x + iy$ such that x and y are real numbers.

Sol -

$z = 2(2-i) + \frac{7-4i}{2+i} - 8$

$z = 4 - 2i + \frac{(7-4i)(2-i)}{5} - 8$

$z = 4 - 2i + \frac{14 - 15i - 8i + 4i^2}{5} - 8$

$z = 4 - 2i + \frac{18 - 23i}{5} - 8$

$z = 4 - 2i + \frac{18}{5} - \frac{23i}{5} - 8$

$z = \frac{14}{5} - \frac{33i}{5}$

Therefore, $z = \frac{14}{5} - \frac{33i}{5}$ in the form of $x + iy$ where

$x = \frac{14}{5}$ and $y = -\frac{33}{5}$

OR

a) Solve the inequality

i) $x^2 + x - 28 < 2$

Sol -

$= x^2 + x - 28 < 2$

$= x^2 + x - 28 - 2 < 0$

$= x^2 + x - 30 < 0$

$= x^2 + 6x - 5x - 30 < 0$

$= x(x+6) - 5(x+6) < 0$

$= (x-5)(x+6) < 0$

$x - 5 < 0 \qquad\qquad x + 6 < 0$

$x < 5 \qquad\qquad x < -6$

ii) $-1 < 4x + 2 < 10$

$4x + 2 < 10 \qquad\qquad 4x + 2 > -1$

$4x < 10 - 2 \qquad\qquad 4x > -1 - 2$

$4x < 8 \qquad\qquad 4x > -3$

$x < 2 \qquad\qquad x > -\frac{3}{4}$

$-\frac{3}{4} < x < 2$

4. Answer any one out of the following questions.

a) In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Sol- The word MISSISSIPPI has 1 M, 4 I's, 4 S's, 2 P's

Total 11 letters.

∴ Number of all type of arrangements possible with given alphabets.

$$\frac{11!}{4!\,4!\,2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2}$$

$$= 34650$$

Let us first find the case when all I's together and so take it as one packet or unit.

We have,
1 M, 1 unit of 4 I's , 4 S's , 2 P's
Total 8 units
∴ number of arrangements possible when all the I's is together

$\frac{8!}{4!\,2!} = 840$

The distinct permutations of the letters of the word MISSISSIPPI when four I's do not come together =
$34650 - 840 = 33810$

b) How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Sol - There are 10 digits from 0 to 9
First place after 67 can be filled in 8 ways.
Second place after 67 can be filled in 6 ways.
Number of telephone numbers that can be constructed =
$8 \times 7 \times 6 = 336$

OR

a) A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

SOLVED ASSIGNMENT ANSWERS
(2023-2024)

Sol- i) Total number of candidates = 2 men + 3 women = 5
Person required in committee = 3
No. of ways = $^5C_3 = 10$

ii) Total men = 2
Total women = 3
No. of ways of selecting 1 men and 2 woman =
$^2C_1 \times {^3C_2} = 2 \times 3 = 6$

b) Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one ace in each combination.

Sol - In a deck of 52 cards, there are 4 aces.
A combination of 5 cards have to be made in which there is exactly one ace.
Then, one ace can be selected in $^4C_1$ ways and the remaining 4 cards can be selected out of 48 cards in $^{48}C_4$ ways.
Required number of 5 card combinations -

$= {^{48}C_4} \times {^4C_1}$

$= \frac{48!}{4!\,(44)!} \times \frac{4!}{1!\,3!}$

$= \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1} \times 4$

$= 778320$

SOLVED ASSIGNMENT ANSWERS
(2023-2024)

5. Answer any one out of the following questions.
Prove the following by using the principle of mathematical induction for all $n \in N$:

a) $1.2 + 2.2^2 + 3.2^2 + \dots + n.2^n = (n-1)2^{n+1} + 2$

sol- Let the given statement be $P(n)$. i.e.,

$P(n) = 1.2 + 2.2^2 + 3.2^2 + \dots + n.2^n = (n-1)2^{n+1} + 2$

$n = 1$ we have

$P(1) = 1.2 = 2 = (1-1)2^{1+1} + 2 = 0 + 2 = 2$

which is true

Let $P(K)$ be true for some positive integer K i.e,

$1.2 + 2.2^2 + 3.2^2 + \dots + K.2^K = (K-1)\,2K^{+1} + 2 \dots (i)$

We shall now prove that $P(K+1)$ is true.

consider,

$\{1.2 + 2.2^2 + 3.2^3 + \dots + K.2^K\} + \{K+1\}.2^{K+1}$

$= (K-1)2^{K+1} + 2 + (K+1)2^{K+1}$

$= 2^{K+1}\{(K-1) + (K+1)\} + 2$

$= 2^{K+1}.2K + 2$

$= K.2^{(K+1)+1} + 2$

$= \{(K+1)-1\}2^{[(K+1)+1]} + 2$

Thus, $P(K+1)$ is true whenever $P(K)$ is true.
Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e, n.

b) $n(n+1)(n+5)$ is a multiple of 3.

sol- Let the given statement be $P(n)$, i.e -

For $n=1$

$P(1) = 1(2)(6) = 12$ which is divisible by 3.

Let $P(K)$ is true

$P(K) = K(K+1)(K+5) = 3m$ where $m \in N$

$\Rightarrow K^3 + 6K^2 + 5K = 3m$

$\Rightarrow K^3 = -6K^2 - 5K + 3m$

Now we will prove that $P(K+1)$ is true; $P(K+1) = (K+1)(K+2)(K+6) = K^3 + 9K^2 + 20$

Putting the value of K^3 in the above equation we get,

$(3m - 6K^2 - 5K) + 9K^2 + 20K + 12$

$= 3m + 3K^2 + 15K + 12$

$= 3(m + K^2 + 5K + 4)$

$3r$ where $r = m + K^2 + 5K + 4$

Since $P(K+1)$ is true whenever $P(K)$ is true.

So, by the principle of induction, $P(n)$ is divisible by 3 for all $n \in N$.

OR

a) Expand $\left(\frac{x}{3} + \frac{2}{y}\right)^4$

Sol - $(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \ldots + {}^nC_n x^{n-n} y^n$

$= {}^4C_0 \left(\frac{x}{3}\right)^4 + {}^4C_1 \left(\frac{x}{3}\right)^{4-1} \left(\frac{2}{y}\right)^1 + {}^4C_2 \left(\frac{x}{3}\right)^{4-2} \left(\frac{2}{y}\right)^2 + {}^4C_3 \left(\frac{x}{3}\right)^{4-3} \left(\frac{2}{y}\right)^3 + {}^4C_4 \left(\frac{2}{y}\right)^4$

$^nC_0 = 1$

$^nC_r = \frac{n!}{n-r\,! \times r!}$

$= \frac{x^4}{3^4} + \frac{4 \times x^3}{3^2} \times \frac{2}{y} + \frac{4!}{2!\,2!} \times \frac{x^2}{3^2} \times \frac{2^2}{y^2} + \frac{4!}{1!\,3!} \times \left(\frac{x}{3}\right)\left(\frac{2^3}{y^3}\right) + \frac{2^4}{y^4}$

$= \frac{x^4}{81} + \frac{8x^3}{27y} + \frac{\cancel{4}^{2} \times \cancel{3!} \times \cancel{2!}}{\cancel{2} \times 1 \times \cancel{2!}} \times \frac{x^2 \times 4}{{}_3\cancel{9} \times y^2} + \frac{4 \times \cancel{3!}}{\cancel{3!}} \times \frac{x}{3} \times \frac{8}{y^3} + \frac{16}{y^4}$

$= \frac{x^4}{81} + \frac{8x^3}{27y} + \frac{8x^2}{3y^2} + \frac{32x}{3y^3} + \frac{16}{y^4}$

b) Find the remainder when 7^{103} is divided by 25.

Sol- $7^{103} = 7.(49)^{51} = 7(50-1)^{51}$

$= 7(50^{51} - {}^{51}C_1\, 50^{50} + \ldots + {}^{51}C_{50}\, 50\} - 7$

$= 7\{50^{51} - {}^{51}C_1\, 50^{50} + \ldots + {}^{51}C_{50}\, 50\} - 25 + 18$

remainder when 7^{103} is divided by 25 is 18.

c) Find the last two digits of the number $(13)^{10}$.

Sol- $(13)^{10} = (169)^5 = (170-1)^5$

$= 5C_0\,(170)^5 - 5C_1\,(170)^4 + 5C_2\,(170)^3 - 5C_3\,(170)^2 + 5C_4\,(170) - 5C_5$

$= 5C_0\,(170)^5 - 5C_1\,(170)^4 + 5C_2\,(170)^3 - 5C_3\,(170)^2 + 5(170) - 1$

A multiple of $100 + 5(170) - 1 = 100K - 849$

∴ The last two digits are 49.

6. Answer any one out of the following questions.

SOLVED ASSIGNMENT ANSWERS (2023-2024)

a) A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	Less than 4000	4000 to 9000	9001 to 14000	More than 14000
Frequency	20	210	325	445

If a tyre is bought from this company, what is the probability that:

i) it has to be substituted before 4000 km is covered?

Sol - Total number of trials = 1000

Frequency of tyre that needs to be replaced before it covers 4000 km = 20

So, P (tyre to be replaced before it covers 4000 km) $= \frac{20}{1000}$

$= 0.02$

ii) it will last more than 9000 km?

Sol - Total number of trials = 1000

Frequency of tyre that it will last more than 9000 km = Frequency of tyre that will last 9000 to 14000 km + Frequency of tyre that will last more than 14000 km

$= 325 + 445 = 770$

So, P (tyre will last more than 9000 km) $= \frac{770}{1000} = 0.77$

iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Sol - Total number of trials = 1000

Frequency of tyre that requires replacement between 4000 & 14000 km = Frequency of tyre that will last between 4000 to 9000 km + Frequency of tyre that will last between 4000 to 14000 km

$= 210 + 325 = 535$

P(tyre requiring replacement between 4000 km and 14000 km) = $\frac{535}{1000} = 0.535$

b) Two players, Sangeet and Rashmi play a tennis match. The probability of Sangeet winning the match is 0.62. What is the probability that Rashmi will win the match?

Sol - Probability of Sangeeta winning the match is 0.62.

P(A) of the random experiment is,

$P(A) = \frac{\text{number of favourable outcome}}{\text{Total number of outcomes}}$

Probability of Rashma winning the match is:

$P(E) = 1 - P$

$P(E) = 1 - 0.62$

$P(E) = 0.38$

Probability of winning the match by Rashmi is 0.38

c) A coin is tossed three times, consider the following events

P : 'No head appears',
Q : Exactly one head appears' and
R : At least two heads appear'.
Check whether they form a set of mutually exclusive and exhaustive events.

Sol- If 3 coins are tossed, possible outcomes are:
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A : No head appears
B : Exactly one head appears
C : At least two head appears

A = TTT
B = HTT, THT, TTH
C = HHT, HTH, THH, HHH

$A \cup B \cup C$ = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} = S

A, B and C are exhaustive events.

$A \cap B = \phi$

A & B are mutually exclusive

$B \cap C = \phi$

B & C are mutually exclusive

$A \cap C = \phi$

A & C are mutually exclusive

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

OR

a) Find the range and coefficient of range of the following data :

(i) 63, 89, 98, 125, 79, 108, 117, 68

sol- The largest value (L) = 125

The smallest value (S) = 63

Range = L-S = 125 - 63 = 62

coefficient of range $= \frac{L-S}{L+S}$

$= \frac{125 - 63}{125 + 63}$

$= \frac{62}{188} = 0.33$

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

sol- Largest value (L) = 61.4

Smallest value (S) = 13.6

Range = L - S

= 61.4 - 13.6 = 47.8

coefficient of range $= \frac{L-S}{L+S}$

$= \frac{61.4 - 13.6}{61.4 + 13.6}$

$= \frac{47.8}{75} = 0.64$

b) If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Sol- Range = 36.8
Smallest value = 13.4
Largest value = L = R + S
= 36.8 + 13.4 = 50.2

c) A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Sol - $\text{Mean} = \frac{\Sigma x}{n}$

$$\text{Mean} = \frac{32 + 35 + 37 + 30 + 33 + 36 + 35 + 37}{8}$$

$$= \frac{275}{8} = 34.375$$

Student	Pages completed	Deviation (X - Mean)
1	32	-2.375
2	35	0.625
3	37	2.625
4	30	-4.375
5	33	-1.375
6	36	1.625
7	35	0.625
8	37	2.625

Square each deviation.

Student	Pages completed	Deviation (X - Mean)	Deviation Squared
1	32	-2.375	5.640625
2	35	0.625	0.390625
3	37	2.625	6.890625
4	30	-4.375	19.140625
5	33	-1.375	1.890625
6	36	1.625	2.640625
7	35	0.625	0.390625
8	37	2.625	6.890625

$\Sigma (X - Mean)^2 = 43.10546875$

$$\text{Variance} = \frac{\Sigma (X - Mean)^2}{n}$$

$$= \frac{43.10546875}{8}$$

$$= 5.38818359375$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{5.38818359375}$$

$$\approx 2.32$$

Therefore, the standard deviation of the pages yet to be completed by the students is approximately 2.32