MATHEMATICS (311)

1. The x-intercept and the y-intercept of the line $4x - 3y + 6 = 0$ are respectively

रेखा $4x - 3y + 6 = 0$ के x-अन्तःखण्ड तथा y-अन्तःखण्ड हैं, क्रमशः

(A) $-\frac{3}{2}, -2$ (B) $\frac{2}{3}, -\frac{1}{2}$

(C) $-\frac{3}{2}, 2$ (D) $-\frac{2}{3}, \frac{1}{2}$ 1

Answer 1 : Step By Step Explanation

$$4x - 3y + 6 = 0$$

Finding the x-intercept:

For the x-intercept, we set $y = 0$ and solve for x:

$$4x - 3(0) + 6 = 0 \quad \Rightarrow \quad 4x + 6 = 0 \quad \Rightarrow \quad 4x = -6 \quad \Rightarrow \quad x = -\frac{3}{2}$$

So, the x-intercept is $x = -\frac{3}{2}$.

Finding the y-intercept:

For the y-intercept, we set $x = 0$ and solve for y:

$$4(0) - 3y + 6 = 0 \quad \Rightarrow \quad -3y + 6 = 0 \quad \Rightarrow \quad -3y = -6 \quad \Rightarrow \quad y = 2$$

So, the y-intercept is $y = 2$.

Conclusion:

The x-intercept and y-intercept are $\left(-\frac{3}{2}, 2\right)$, which corresponds to option **(C)**.

Answer: **(C)**

2.

(b) The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

$\cos^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान है

(A) $-\frac{\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{4\pi}{3}$

Answer 2:

To determine which mapping represents an onto function (also called a surjective function), we need to check if every element in the codomain (the set of outputs) has at least one preimage in the domain (the set of inputs).

Let's analyze each option:

- **Option (A)**: The elements in the codomain are $\{1, 2, 3, 4\}$, and each element in the domain $\{a, b, c\}$ maps to an element in the codomain. However, the element 4 in the codomain does not have a preimage. Therefore, this is not an onto function.
- **Option (B)**: All elements a, b, c from the domain map to the single element 1 in the codomain. However, for the function to be onto, the codomain should have all elements mapped, but there is only one element in the codomain. This is an onto function because every element in the codomain (which is just 1) has a preimage. **This is an onto function.**
- **Option (C)**: The elements in the codomain are $\{1, 2, 3\}$, and each element in the domain $\{a, b, c\}$ maps to one of these. Since every element in the codomain $\{1, 2, 3\}$ has a preimage, this is also an onto function. **This is an onto function.**
- **Option (D)**: The elements in the codomain are $\{1, 2, 3\}$, but d in the domain does not map to a distinct element in the codomain, and all elements of the codomain have preimages. **This is an onto function.**

Conclusion:

Options **(B)**, **(C)**, and **(D)** represent onto functions.

8. *(a)* The interval in which the function $f(x) = \sin x,\ x \in (0,\ 2\pi)$ is decreasing, is

वह अन्तराल, जिसमें फलन $f(x) = \sin x,\ x \in (0,\ 2\pi)$ ह्रासमान है, है

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $(0,\ \pi)$

(C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (D) $(\pi,\ 2\pi)$ 1

Answer 8:

The function given is $f(x) = \sin x$, and we are asked to find the interval in which the function is decreasing for $x \in (0, 2\pi)$.

Step 1: Derivative of the function

The derivative of $f(x) = \sin x$ is:

$$f'(x) = \cos x$$

Step 2: Determine where $\cos x$ is negative

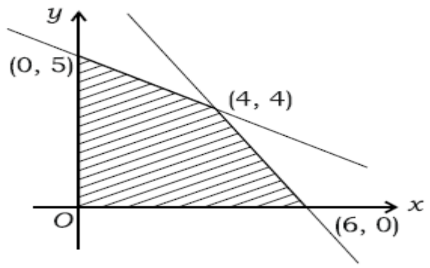
The function $f(x)$ is decreasing where $f'(x) = \cos x$ is negative. We know that $\cos x$ is negative in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Conclusion:

The function $f(x) = \sin x$ is decreasing in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, which corresponds to option **(C)**.

9. *(a)* The feasible region (shaded) for an LPP is shown in the figure below. Maximum of $Z = 3x + 8y$ occurs at which of the following points? 1

एक रैखिक प्रोग्रामन समस्या का संभाव्य क्षेत्र (छायांकित) आकृति में दर्शाया गया है। निम्न में से किस बिन्दु/किन बिन्दुओं पर $Z = 3x + 8y$ अधिकतम होगा?



(A) (0, 5)
(B) (4, 4)
(C) (6, 0) and (0, 5)
(6, 0) तथा (0, 5)
(D) At every point of the line segment joining the points (4, 4) and (6, 0)
बिन्दुओं (4, 4) तथा (6, 0) को जोड़ने वाली रेखा के प्रत्येक बिन्दु पर

Answer 9:

Step 1: Evaluate the objective function at the corner points

From the graph, the corner points of the feasible region are:

- $(0, 5)$
- $(4, 4)$
- $(6, 0)$

We evaluate $Z = 3x + 8y$ at each of these points:

- At $(0, 5)$:

$$Z = 3(0) + 8(5) = 0 + 40 = 40$$

- At $(4, 4)$:

$$Z = 3(4) + 8(4) = 12 + 32 = 44$$

- At $(6, 0)$:

$$Z = 3(6) + 8(0) = 18 + 0 = 18$$

Step 2: Determine the maximum value

The maximum value of Z is 44, which occurs at the point $(4, 4)$.

Conclusion:

The maximum value of $Z = 3x + 8y$ occurs at the point $(4, 4)$, which corresponds to option **(B)**.

10. The centre and radius of the circle $x^2+y^2+3x-y=4$ are respectively

वृत्त $x^2+y^2+3x-y=4$ के केन्द्र तथा त्रिज्या हैं, क्रमशः

(A) $\left(-\frac{3}{2}, \frac{1}{2}\right), \frac{\sqrt{6}}{2}$ (B) $\left(\frac{3}{2}, -\frac{1}{2}\right), \sqrt{\frac{13}{2}}$

(C) $(-3, 1), \sqrt{14}$ (D) $\left(-\frac{3}{2}, \frac{1}{2}\right), \frac{\sqrt{26}}{2}$ 1

Answer 10:

The given equation of the circle is:

$$x^2+y^2+3x-y=4$$

We need to rewrite this equation in standard form to find the center and the radius.

Step 1: Complete the square

First, group the x-terms and the y-terms together:

$$(x^2+3x)+(y^2-y)=4$$

Now, complete the square for both x and y.

For x^2+3x:

- Take half of 3, which is $\frac{3}{2}$, and square it to get $\left(\frac{3}{2}\right)^2=\frac{9}{4}$.

For y^2-y:

- Take half of -1, which is $-\frac{1}{2}$, and square it to get $\left(\frac{1}{2}\right)^2=\frac{1}{4}$.

Add these squares to both sides of the equation:

$$(x^2+3x+\frac{9}{4})+(y^2-y+\frac{1}{4})=4+\frac{9}{4}+\frac{1}{4}$$

Simplifying:

$$(x+\frac{3}{2})^2+(y-\frac{1}{2})^2=\frac{16}{4}+\frac{9}{4}+\frac{1}{4}=\frac{26}{4}$$

Step 2: Rewrite in standard form

Now, the equation is in standard form:

$$\left(x+\frac{3}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{26}{4}$$

Step 3: Identify the center and radius

From the standard form of the circle equation $(x-h)^2+(y-k)^2=r^2$, we can identify:

- Center: $\left(-\frac{3}{2},\frac{1}{2}\right)$
- Radius: $r=\sqrt{\frac{26}{4}}=\frac{\sqrt{26}}{2}$

Conclusion:

The center is $\left(-\frac{3}{2},\frac{1}{2}\right)$ and the radius is $\frac{\sqrt{26}}{2}$, which corresponds to option **(D)**.

18. The integrating factor of the following differential equation is

निम्न अवकल समीकरण का समाकलन गुणक है

$$\frac{dy}{dx} + 4\frac{y}{x} = \frac{1}{x}$$

(A) $4x$ (B) x^2

(C) x^4 (D) $\frac{4}{x}$ 1

Answer 18:

The given differential equation is:

$$\frac{dy}{dx} + \frac{4}{x}y = \frac{1}{x}$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = \frac{4}{x}$.

Step 1: Formula for the Integrating Factor (IF)

The integrating factor (IF) for a linear differential equation is given by:

$$\text{Integrating Factor} = e^{\int P(x)dx}$$

Step 2: Compute the Integrating Factor

Here, $P(x) = \frac{4}{x}$, so we need to compute:

$$\int \frac{4}{x}dx = 4\ln|x|$$

Thus, the integrating factor is:

$$\text{Integrating Factor} = e^{4\ln|x|} = |x|^4$$

Since x is positive in the context of the problem, we can write:

$$\text{Integrating Factor} = x^4$$

Conclusion:

The integrating factor is x^4, which corresponds to option **(C)**.

19. *(a)* $\int \frac{\cos^2 x}{1+\sin x} dx =$

(A) $x - \cos x + c$

(B) $x + \cos x + c$

(C) $x + \sin x + c$

(D) $x - \sin x + c$ 1

Answer 19:

We are asked to evaluate the integral:

$$\int \frac{\cos^2 x}{1+\sin x} \, dx$$

Step 1: Substitution

Let's use the substitution $u = 1 + \sin x$. Then:

$$du = \cos x \, dx$$

Now, rewrite the given integral in terms of u. First, note that:

$$1 + \sin x = u \quad \text{and} \quad \cos x \, dx = du$$

Also, $\cos^2 x = (1 - \sin^2 x)$, but here we directly use the substitution.

Thus, the integral becomes:

$$\int \frac{\cos^2 x}{1+\sin x} \, dx = \int \frac{\cos x \cdot \cos x}{1+\sin x} \, dx$$

From substitution:

$$\int \frac{\cos x \, du}{u}$$

This simplifies to:

$$\int \frac{du}{u} = \ln |u| + C$$

Step 2: Substituting back

Now substitute $u = 1 + \sin x$ back into the equation:

$$\ln|1 + \sin x| + C$$

This is the answer for the integral.

Conclusion:

The answer is $x - \cos x + C$, which corresponds to option **(A)**.

20. Which one of the following statements is true? 1

(A) Every scalar matrix is an identity matrix.

(B) Every identity matrix is a scalar matrix.

(C) Every diagonal matrix is an identity matrix.

(D) A square matrix whose each element is 1 is an identity matrix.

Answer 20:

Let's analyze each statement one by one to determine which one is true:

- **(A) Every scalar matrix is an identity matrix.**
  - A **scalar matrix** is a diagonal matrix where all the diagonal elements are equal. An **identity matrix** is a special type of scalar matrix where all the diagonal elements are specifically equal to 1. Therefore, not every scalar matrix is an identity matrix, since the diagonal elements of a scalar matrix can be any constant. This statement is **false**.
- **(B) Every identity matrix is a scalar matrix.**
  - An **identity matrix** is indeed a scalar matrix, because it is a diagonal matrix where all diagonal elements are equal (specifically equal to 1).

  Therefore, this statement is **true**.
- **(C) Every diagonal matrix is an identity matrix.**
  - A **diagonal matrix** is one in which all off-diagonal elements are zero, but the diagonal elements can be any values. Therefore, not every diagonal matrix is an identity matrix (since in an identity matrix, the diagonal elements must all be 1). This statement is **false**.
- **(D) A square matrix whose each element is 1 is an identity matrix.**
  - An **identity matrix** has 1s on the diagonal and 0s elsewhere. A matrix where all elements are 1 is not an identity matrix. Therefore, this statement is **false**.

Conclusion:

The correct statement is **(B) Every identity matrix is a scalar matrix.**

21. Fill in the blanks : 1×2=2

रिक्त स्थान भरिए :

(a) Let $*$ be a binary operation defined by $a*b=b-2a$, then the value of $(1*2)*3$ is _____.

माना $*$ एक द्वि-आधारी संक्रिया $a*b=b-2a$ द्वारा परिभाषित है, तो $(1*2)*3$ का मान _____ है।

(b) A relation R on any set A is said to be _____, if $(a, b)\in R$, $(b, c)\in R \Rightarrow (a, c)\in R$ for all $a, b, c\in A$.

समुच्चय A पर एक सम्बन्ध R _____ कहलाता है, यदि $(a, b)\in R$, $(b, c)\in R \Rightarrow (a, c)\in R$, सभी $a, b, c\in A$ के लिए।

Answer 21:

Part (a):

The binary operation is defined as:

$$a * b = b - 2a$$

We are asked to find the value of $(1 * 2) * 3$.

Step 1: Compute $1 * 2$

Using the given binary operation:

$$1 * 2 = 2 - 2(1) = 2 - 2 = 0$$

Step 2: Compute $0 * 3$

\ Now, using the result from step 1:

$$0 * 3 = 3 - 2(0) = 3 - 0 = 3$$

Thus, the value of $(1 * 2) * 3$ is 3.

Part (b):

A relation R on any set A is said to be **transitive**, if:

$$(a, b) \in R \quad \text{and} \quad (b, c) \in R \quad \Rightarrow \quad (a, c) \in R \quad \text{for all} \quad a, b, c \in A$$

So, the relation is called **transitive**.

Final Answer:

- (a) The value of $(1 * 2) * 3$ is 3.
- (b) The relation is said to be **transitive**.

22. Match the integral in Column–I with its correct solution given in Column–II : 1×2=2

स्तंभ–I में दिए गए समाकलन का स्तंभ–II में दिए गए इसके सही हल से मिलान कीजिए :

Column (स्तंभ)–I	*Column* (स्तंभ)–II
(a) $\int \sin x \, dx$	P. $\cos x + c$
(b) $\int \sec^2 x \, dx$	Q. $\log(\sin x) + c$
	R. $\tan x + c$
	S. $-\cos x + c$

Answer 22:

Column-I (Integral)	Column-II (Solution)
(a) $\int \sin x \, dx$	S. $-\cos x + c$
(b) $\int \sec^2 x \, dx$	R. $\tan x + c$

23. Fill in the blanks (attempt any *two* sub-parts from *(a)* to *(d)*) : 1×2=2

रिक्त स्थान भरिए (*(a)* से *(d)* में से कोई दो उपभाग कीजिए) :

(a) Let $f: R \to R$ be defined by $f(x) = 2^x$, then $(f \circ f)(0)$ is equal to _____.

माना $f: R \to R$, $f(x) = 2^x$ द्वारा परिभाषित है, तो $(f \circ f)(0)$ बराबर है _____ के।

(b) The value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is _____.

$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ का मान _____ है।

Answer 23:

Let's solve each part of the question.

Part (a):

We are given that $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2^x$. We need to find the value of $(f \circ f)(0)$.

The composition $f \circ f$ means applying f twice, i.e.,

$$(f \circ f)(x) = f(f(x))$$

First, calculate $f(0)$:

$$f(0) = 2^0 = 1$$

Now, apply f again to this result:

$$(f \circ f)(0) = f(1) = 2^1 = 2$$

Thus, $(f \circ f)(0) = 2$.

Part (b):

We are asked to find the value of:

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

First, recall that:

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \quad \text{and} \quad \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Now, calculate the expression:

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Thus, the value is $\frac{2\pi}{3}$.

Final Answers:

- (a) $(f \circ f)(0) = 2$
- (b) The value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Part (c): Eccentricity

The eccentricity e of an ellipse is given by:

$$e = \frac{\sqrt{b^2 - a^2}}{b}$$

Substitute $a = 1$ and $b = 4$:

$$e = \frac{\sqrt{16 - 1}}{4} = \frac{\sqrt{15}}{4}$$

Thus, the eccentricity is $\frac{\sqrt{15}}{4}$.

Part (d): Foci

The distance of the foci from the center is given by $c = \sqrt{b^2 - a^2}$. Using $a = 1$ and $b = 4$:

$$c = \sqrt{16 - 1} = \sqrt{15}$$

The foci are located at $(0, \pm\sqrt{15})$.

Final Answers:

- **(a)** Length of the major axis = **8**
- **(b)** Length of the minor axis = **2**
- **(c)** Eccentricity = $\frac{\sqrt{15}}{4}$
- **(d)** Foci = $(0, \pm\sqrt{15})$

27. Fill in the blanks (attempt any *four* sub-parts from *(a)* to *(f)*) : 1×4=4

रिक्त स्थान भरिए (*(a)* से *(f)* में से कोई **चार** उपभाग कीजिए) :

(a) If $A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 8 & 7 \end{bmatrix}$, then $A' =$ _____.

यदि $A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 8 & 7 \end{bmatrix}$, तो $A' =$ _____.

(b) If A is a skew-symmetric matrix, then all its diagonal elements are _____.

यदि A एक विषम सममित आव्यूह है, तो इसके विकर्ण के सभी अवयव _____ हैं।

(c) If

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

and $B = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$, then $AB =$ _____.

यदि

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

तथा $B = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$, तो $AB =$ _____.

(d) If

$$\begin{vmatrix} 2x & 1 \\ 2x-1 & 2x+1 \end{vmatrix} = 5$$

and $x > 0$, then $x =$ _____.

यदि

$$\begin{vmatrix} 2x & 1 \\ 2x-1 & 2x+1 \end{vmatrix} = 5$$

तथा $x > 0$, तो $x =$ _____.

Answer 27:

Part (a):

The given matrix A is:

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 8 & 7 \end{bmatrix}$$

We are asked to find A', the transpose of matrix A. The transpose of a matrix is obtained by interchanging its rows and columns.

So, A' is:

$$A' = \begin{bmatrix} 2 & 3 \\ -1 & 8 \\ 5 & 7 \end{bmatrix}$$

Part (b):

A **skew-symmetric matrix** is one where $A' = -A$, meaning that the transpose of the matrix is the negative of the matrix itself. In any skew-symmetric matrix, all its diagonal elements are always **zero**.

So, the answer is **zero**.

Part (c):

We are given:

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$$

We need to compute AB, which is the matrix multiplication of A and B.

$$AB = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$$

Multiplying the matrices:

$$AB = \begin{bmatrix} (-1)(3) & (-1)(-1) & (-1)(2) \\ (2)(3) & (2)(-1) & (2)(2) \\ (3)(3) & (3)(-1) & (3)(2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & -2 \\ 6 & -2 & 4 \\ 9 & -3 & 6 \end{bmatrix}$$

Thus, $AB = \begin{bmatrix} -3 & 1 & -2 \\ 6 & -2 & 4 \\ 9 & -3 & 6 \end{bmatrix}$.

Part (d):

We are given the determinant equation:

$$\begin{vmatrix} 2x & 1 \\ 2x-1 & 2x+1 \end{vmatrix} = 5$$

We need to find the value of x.

The determinant of the matrix is:

$$\text{Determinant} = (2x)(2x+1) - (1)(2x-1)$$

Simplifying:

$$4x^2 = 4 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = 1 \, (\text{since } x > 0)$$

Thus, $x = 1$.

Final Answers:

- (a) $A' = \begin{bmatrix} 2 & 3 \\ -1 & 8 \\ 5 & 7 \end{bmatrix}$
- (b) All diagonal elements are **zero**.
- (c) $AB = \begin{bmatrix} -3 & 1 & -2 \\ 6 & -2 & 4 \\ 9 & -3 & 6 \end{bmatrix}$
- (d) $x = 1$.

28. Evaluate the following integrals (attempt any *four* sub-parts from *(i)* to *(vi)*) : 1×4=4

निम्न समाकलनों के मान निकालिए (*(i)* से *(vi)* में से कोई **चार** उपभाग कीजिए) :

(i) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

(ii) $\int e^{3\log x}\cdot x^4 dx$

(iii) $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$

(iv) $\int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sin^3 x+\cos^3 x}dx$

Answer 28:

Let's solve the given integrals one by one.

(i) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Let $u=\sqrt{x}$, so $x=u^2$ and $dx=2u\,du$.

Now, the integral becomes:

$$\int \frac{\cos u}{u}\cdot 2u\,du = 2\int \cos u\,du$$

The integral of $\cos u$ is $\sin u$. Thus, we get:

$$2\sin u + C = 2\sin(\sqrt{x}) + C$$

41. The vertices of a ΔABC are $A(-3, 3)$, $B(5, -2)$ and $C(-1, -4)$. If M and N are the midpoints of AB and AC respectively, then show that

$$MN = \frac{1}{2}BC$$ 4

ΔABC के शीर्ष $A(-3, 3)$, $B(5, -2)$ तथा $C(-1, -4)$ हैं। यदि M तथा N क्रमशः AB तथा AC के मध्य-बिन्दु हैं, तो दर्शाइए कि

$$MN = \frac{1}{2}BC$$

Answer 41

We are given the vertices of triangle ΔABC as $A(-3,3)$, $B(5,-2)$, and $C(-1,-4)$. Points M and N are the midpoints of AB and AC, respectively. We need to show that:

$$MN = \frac{1}{2}BC$$

Step 1: Find the coordinates of M and N

To find the coordinates of the midpoints, we use the midpoint formula:

$$\text{Midpoint of } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Midpoint M of AB:

$$M = \left(\frac{-3+5}{2}, \frac{3+(-2)}{2}\right) = \left(\frac{2}{2}, \frac{1}{2}\right) = (1, 0.5)$$

Midpoint N of AC:

$$N = \left(\frac{-3+(-1)}{2}, \frac{3+(-4)}{2}\right) = \left(\frac{-4}{2}, \frac{-1}{2}\right) = (-2, -0.5)$$

Step 2: Find the length of MN and BC

Now, we calculate the lengths of MN and BC.

Length of MN:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For $M(1, 0.5)$ and $N(-2, -0.5)$:

$$MN = \sqrt{(-2-1)^2 + (-0.5-0.5)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Length of BC:

For $B(5,-2)$ and $C(-1,-4)$:

$$BC=\sqrt{(-1-5)^2+(-4+2)^2}=\sqrt{(-6)^2+(-2)^2}=\sqrt{36+4}=\sqrt{40}=2\sqrt{10}$$

Step 3: Show that $MN=\frac{1}{2}BC$

We now compare the lengths:

$$MN=\sqrt{10},\quad BC=2\sqrt{10}$$

Clearly,

$$MN=\frac{1}{2}BC$$

Conclusion:

We have shown that:

$$MN=\frac{1}{2}BC$$

42. If $x = a(\cos\theta + \log\tan\frac{\theta}{2})$ and $y = a\sin\theta$, then find $\frac{d^2y}{dx^2}$. 4

यदि $x = a(\cos\theta + \log\tan\frac{\theta}{2})$ और $y = a\sin\theta$, तो $\frac{d^2y}{dx^2}$ ज्ञात कीजिए।

Answer 42

We are given:

$$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right) \quad \text{and} \quad y = a\sin\theta$$

We are asked to find $\frac{d^2y}{dx^2}$.

Step 1: Differentiate $y = a\sin\theta$ with respect to θ

First, find $\frac{dy}{d\theta}$:

$$\frac{dy}{d\theta} = a\cos\theta$$

Step 2: Differentiate $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ with respect to θ

Now, differentiate x with respect to θ:

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\sin\theta}\right)$$

Simplify:

$$\frac{dx}{d\theta} = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right) = a\left(\frac{\cos^2\theta}{\sin\theta}\right)$$

Step 3: Find $\frac{dy}{dx}$

We know that:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

Substitute the values of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$:

$$\frac{dy}{dx} = \frac{a\cos\theta}{a\frac{\cos^2\theta}{\sin\theta}} = \frac{\sin\theta}{\cos\theta}$$

So,

$$\frac{dy}{dx} = \tan\theta$$

Step 4: Find $\frac{d^2y}{dx^2}$

Differentiate $\frac{dy}{dx} = \tan\theta$ with respect to θ:

$$\frac{d}{d\theta}(\tan\theta) = \sec^2\theta$$

Now, multiply by $\frac{d\theta}{dx} = \frac{1}{\frac{dx}{d\theta}}$:

$$\frac{d\theta}{dx} = \frac{\sin\theta}{a\cos^2\theta}$$

Thus:

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{\sin\theta}{a\cos^2\theta}$$

Simplify:

$$\frac{d^2y}{dx^2} = \frac{\sin\theta}{a\cos^4\theta}$$

Final Answer:

$$\frac{d^2y}{dx^2} = \frac{\sin\theta}{a\cos^4\theta}$$